# **EMPIRICALLY BOUNDARY SHEAR STRESS COUPLED WITH EVOLUTIONARY ALGORITHMS IN A V-SHAPED CHANNEL**

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**Abstract:** To understand the lateral distribution of boundary shear stress, a series of measurements was undertaken in the uniform flow condition at two different bed slopes of 0.1%, 0.2% in a V-shaped bottom channel and the distribution results for the mild slope and steeper channels is discussed. In addition, the performance of GEP model, as an evolutionary algorithm was evaluated to predict the shear stress distribution in the channel by varying the bed slope. Using the common different error criteria, the results demonstrate a successful application of the evolutionary algorithms in developing the accuracy level of predicting the boundary shear stress. Beside these results, it is proved that GEP is a powerful tool for pattern recognition and data interpretation, as expressed an explicit and simple predictive equation of boundary shear stress distribution along the wetted perimeter of the channel that can be used by anyone not special.

KEY WORDS: boundary shear stress, evolutionary algorithms, V-shaped channel, explicit equation.

### **1. INTRODUCTION**

In open channel flow, it is possible to derive a simple formula that allows evaluation of the mean boundary shear stress acting over the wetted perimeter of the channel. In order to understand the general 3D flow structures, however the lateral distribution of boundary shear stress needs to be explored. The well-known Navier-Stokes equations have already been used for extraction of turbulent flow structures, where express the mean and fluctuating velocity components. In this case, the boundary shear stress,  $\tau_b$ , has an important role. The pattern and number of secondary flow cells as well as channel cross sectional shape will influence the lateral distribution of boundary shear stress. Einstein (1942) developed the first method to estimate mean shear stresses at the bed and at the walls in an open channel. Meyer-Peter and Müller (1948) presented a similar method to Einstein's, without making any reference to it. Taylor (1961) concluded

that the Einstein methodwas appropriate to evaluate friction with an aspect ratio smaller than 0.5. Johnson (1942) admitted the convenience of using the friction logarithmic law with Einstein's method.Vanoni & Brooks (1957) refined Johnson's method and explained how to separate bed and wall shear stresses. The ASCE (1975) recommended to using the Vanoni & Brooks method, warning about possible deficiencies in the estimation of the friction factor. ASCE, however advises about making direct measurements to obtain true values of shear stress at the bed and at the walls since there is not any experimental support in Vanoni & Brooks's method. Many researches have considered the shear distribution problem by obtaining local shear stress as a fraction of the total shear stress and as a function of the aspect ratio B/h, where B is the channel width and h is the flow depth, e.g. and Mohammadi (2000 & 2002). These authors studied smooth channels, separately from rough channels. In this paper, to investigate the hydraulic characteristics of a V-shaped bottom channel, several series of experimentswere conducted for measuring boundary shear stress aroundwetted perimeters (see Fig. 1 for channel cross-section).



Figure 1. Geometry of the V-shaped channel and notation: B=460 mm,  $\Delta h = 50 \text{ mm}$ ;  $h_{max} = 300 \text{ mm}$ .

A Preston tube was used for measuring dynamic pressures to evaluate boundary shear stress and shear force. For a certain channel bed slope, a discharge was introduced and uniform flow was established using stage-discharge results and discharge-tailgate relationships. For every set of flows both point velocities in cross section and dynamic pressures in contact with channel boundary were measured at the same flow condition. For both velocity and boundary shear stress, the data are analyzed from a variety of different perspectives, and the results from each analysis are used to interpret the mechanics occurring in the flow. The findings from each perspective complement each other and highlight the consistency of the experimental data. It is intended to verify that the experimental data related to shear stress show similar results for open channels and rectangular ducts if they keep geometric similitude. It is also intended to find the influence of cross sectional shape on shear stresses at the boundary. Finally, the paper deals with the presentation of some existing experimental data on boundary shear stress, together with analysis of the results obtained so far. The associated discussions will also be presented.

#### 2. EXPERIMENTAL APPARATUS AND PROCEDURE

To investigate the hydraulic characteristics of a V-shaped bottom channel, several series of experiments were conducted for measuring boundary shear stress around wetted perimeters. The experimental channels were built inside the existing 15 m long tilting flume. A Preston tube was used for measuring dynamic pressures to evaluate boundary shear stress and shear force. For a certain channel bed slope, a discharge was introduced and uniform flow was established using stage discharge

results and discharge-tailgate relationships. The flume was supported by two hydraulic jacks and rotated about a hinge joint beneath the middle of the channel. The flume also had a motorized slope control system with a mechanical visual read out on a ruler at the upstream end of the flume used for determining the precise channel bed slope. The maximum slope obtain able was S0 = 2%. The experimental channels, with a V-shaped bottom cross section built by using PVC panels to make a 14.5m long channel having 50mm cross fall, were built along the inside centerline of the existing flume. Water was supplied to the channel by an overhead tank through a 101.6mm pipeline for discharges up to 30 l/s and a 355.6mm pipeline for discharges higher than 30 l/s. To reduce largescale disturbances, and in order to ensure that the flow was uniformly distributed, a system of honeycombing was placed at the upstream end of the channel where the entrance tank and bell-mouth shaped inlet transition section were located. However for the case of supercritical flow i.e. Fr>1 the honeycomb was not very useful. Individual bell-mouth shaped transition sections were designed and made for each channel types and served to reduce separation and improve the development of the mean flow into the channels. Discharge measurements (up to 30 1/s) were made by means of a Venturi meter connected to mercury and air/water manometers at the head of the flume. An electromagnetic flow meter was also installed in the supply line after the Venturi and was used to check discharges. For the case of higher discharges a dall-tube connected to an air/water manometer was used in the 355.6mm diameter supply line. The flume had a very rigid bottom designed for high loads, and therefore it was not necessary to do any deflection tests. A slatted tailgate weir was installed in the downstream end of the channel in order to minimize upstream disturbance of the flow, and hence allowed a greater reach of the channel to be employed for experimental measurement in subcritical flows. The test section consisted 12m long zone, commencing at a distance of 1.25m from the channel entrance and 1.85 m from the flume entrance. However, for supercritical flows, because of the S2 profiles, the test length was reduced to about 7 m. A trolley was mounted on rails running along the flume with a depth probe, having an electrical contact to the water surface level (accurate to 0.1 mm) and hence the channel bed slope was obtained. It has also been possible to do lateral measurements using the

same trolley. The depth was measured at 1m and sometimes half a meter intervals in the test length by means of a centerline pointer probe moved down from the instrument carriage. The present research work deals with the boundary shear stress measured around the wetted perimeter. Local boundary shear stress was measured using the Preston tube technique with a probe having 4.705mm outer diameter. The tube was mounted on a carriage and aligned vertically near the walls and normal to the bed. It was also placed on the channel boundary every 10 mm intervals on the vertical walls and every 20 mm intervals on the bed in the span wise direction. The total pressure arising from the difference between the static and dynamic pressures were recorded by connecting the tube to a simple manometer inclined at 12.52° to the horizontal. The static pressure was measured separately using a Pitot static tube at the centerline of the measuring section, and at least 5 minutes allowed to achieve an accurate reading.

#### **3. PERFORMANCE EVALUATION**

This study adopted a multi- criterion approach (RMSE, MAE, and R) in evaluating the overall performances of the adopted models, as each of the performance indicators provide different information about the predictive ability of the model. The performance indicators RMSE, MAE and R are calculated using equations (1), (2), and (3), respectively.

$$RMSE = \sqrt{\frac{1}{N} \cdot \sum \left(n_i - n_i'\right)^2} \tag{1}$$

$$MAE = \frac{1}{N} \sum \left| n_i - n'_i \right| \tag{2}$$

$$R = \frac{\sum_{i=1}^{n} (n_i - \overline{n}_i)(n'_i - \overline{n}'_i)}{\sqrt{\sum_{i=1}^{n} (n_i - \overline{n}_i)^2 \cdot \sum_{i=1}^{n} (n_i - \overline{n}'_i)^2}}$$
(3)

Where,  $n_i$  and  $n_i$ ' represents the measured and computed Manning coefficients values,  $\overline{n}_i$  and  $\overline{n}'_i$  represents the mean of the measured and computed Manning coefficients values, respectively, and N represents the number of data instances.

## 4. IMPLEMENTATION OF THE GEP MODELS

GP, a branch of the genetic algorithm (GA), is a method for learning the most "fit" computer programs by means of artificial evolution and GEP is an extension to GP that evolves computer programs of different sizes and shapes encoded in linear chromosomes of fixed length. There are five major steps in preparing to use GEP. The first is to choose the fitness function. For this problem, the fitness, fi, ofan individual program, i , is defined by the following expression:

$$f_{i} = \sum_{j=1}^{C_{t}} (M - |C_{(i,j)} - K_{j}|)$$
(4)

Where M is the range of selection, C(i,j) the value returned by the individual chromosome i for fitness case j (out of Ct fitness cases) and Kj is the target value for fitness case j. If  $|C_{(i,j)} - K_j|$  (The precision) less or equal to 0.01, then the precision is equal to zero, andfi = fmax =Ct M. For present study, M= 100 and, therefore, fmax= 1000. The advantage of this kind of fitness function is that the system can find the optimal solution for itself (Ferreeira 2001a, b). Second, the set of terminals T and the set of functions F are chosen to create the chromosomes. In this problem, the terminal set consists obviously of single independent variable (P; wetted perimeter). The choice of the appropriate function set is not so obvious; however, a good guess can always be helpful in order to include all the necessary functions. In this study, four basic arithmetic operators (+, -, \*, /) and some basic mathematical functions ( $\sqrt{1}$ , x2, x3 ,atan, ...) were utilized. The third major step is to choose the chromosomal architecture, i.e., the length of the head and the number of genes. By the head size h=7 and 3 genes per chromosome for each GEP model were recorded to give the best results by evolving the GEP model for 20 000 runs. The fourth major step is to choose the linking function. In this case, sub-ETs linked by addition to obtain a simple and explicit equation. And finally, the fifth major step is to choose the set of genetic operators that cause variation and their rates.

#### **5. RESULTS AND DISCUSIONS**

Different input sets (terminals) are shown in table 1 that includes the local boundary shear stress and boundary shear stress distributions adjusted to the mean energy slope. Run of each model (consist of three objective function and four function sets for both of S=0.1% and S=0.2% or totally 24 models) is done numerous times for all of data. Also table 1 sums up the global average statistical parameters obtained for the GEP approach runs. Tadj is the boundary shear stress distributions adjusted to the mean energy slope, P and R shows the perimeter and hydraulic radius of the channel respectively.

Objective function		Terminals or independent variables	Performance Criteria		
			MAE	R	RMSE
S=0.1%	GP1 ; $\tau(\text{local}) = f(P)$	Р	0.049	0.91	0.065
S=0.1%	GP2; $\tau/\rho gRS_0 = f(P)$	Р	0.058	0.98	0.077
S= 0.1%	GP3 ; tadj= f(P)	Р	0.049	0.83	0.063
S=0.2%	GP4 ; $\tau(\text{local}) = f(P)$	Р	0.029	0.93	0.049
S=0.2%	GP5 ; $\tau/\rho gRS_0 = f(P)$	Р	0.071	0.93	0.09
S= 0.2%	GP6 ; $\tau adj = f(P)$	р	0.024	0.96	0.052

Table 1: Different GP input sets (or terminals) and results of GP-f4 performance

Figure 2 presents the performance parameters of the GEP models split up per slope set. As it is clear in all the three parts of figure 2, an increase in the slope rate causes shear stress to be enhanced along the wetted perimeter of the channel. The trend of GEP predicted models to the observed rates in figure 2 verifies the obtained evaluation criteria in table 1. So that the exact trend of GEP model to the observed rates in figure 2a demonstrates the prediction power of  $\tau_{adj}$  in comparison to the other sets along the wetted perimeter of the channel. For the proposed models, table 2 ranks the evaluation criteria separately and in the last columns, these criteria are ranked for all of the models in comparison with each other. According to the table 2,  $\tau_{adj}$ ,  $\tau(local)$ ,  $\tau/\rho gRS_0$ , are ranked in robustness of prediction. For the best model (GP6-F4) in the table 2, the relation between  $\tau_{adj}$  and wetted perimeter explicitly can be expressed by the equation 5 that is extracted from the GEP model.

 $\tau_{adj} = 1.37(0.12P-66.2)^{-1} + atan(4P^*e^{-P}-0.58)^3 + (0.28P+0.58)^{0.5}$ (5)

### 6. CONCLUSIONS

Boundary shear stress measurements have been undertaken in the uniform flow condition at two different bed slopes of 0.1%, 0.2% in a V-shaped bottom channel. In addition, the performance of GEP model, as an evolutionary algorithm were evaluated to predict the shear stress in the channel by varying the bed slope. A common application of the different error criteria is applied. This study demonstrates a successful application of the evolutionary algorithms in developing the accuracy level of predicting the Boundary shear stress. Beside these results, GEP is a powerful tool for pattern recognition and data interpretation and can give explicit and simple predictive equation that can be used by

anyone not specialized with the GEP technique. The proposed GEP formulae give a practical way for Boundary shear stress to obtain accurate results and encourage the use of GEP as a superior to other proposed intelligent approaches in other aspects of water engineering studies.



Figure 2: GEP models in comparison with (a)  $\tau_{adj}$ , (b)  $\tau$ (local) and (c)  $\tau$ /pgRS<sub>0</sub> along the wetted perimeter of channel

Table 2: Ranking for different GP input set results

		1			
Objective function		Ranking			
					Global Ranking
		MAE	R	RMSE	
S=0.1%	GP1 ; $\tau(\text{local}) = f(P)$	3	4	4	4
S=0.1%	GP2; $\tau/\rho gRS_0 = f(P)$	4	1	5	3
S=0.1%	GP3 : $\tau adi = f(P)$	3	5	3	4
	,	-	-	-	
S=0.2%	GP4 ; $\tau(\text{local}) = f(P)$	2	3	1	2
S=0.2%	GP5 ; $\tau/\rho gRS_0 = f(P)$	5	3	6	5
S=0.2%	GP6 ; $\tau adj = f(P)$	1	2	2	1

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